Modelling of vertical temperature gradient with displacement ventilation

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SUMMARY
Vertical temperature gradient prediction is essential for designing of displacement ventilation airflow rate. Several simplified nodal models were developed to estimate the temperature stratification in rooms with displacement ventilation. Typically, the temperature gradient is modelled to be linear over the room height. The simplified multi-nodal models allow different slopes for the temperature profile between the nodes and predict the accurate temperature gradient for many buoyant flow elements. However, all the current models failed to estimate the temperature gradient in a room with heat loads from window and high-level heat gains. The novel four-nodal model was proposed to calculate the temperature gradient in a room with various single flow elements and combinations of them. The measurement data was compared with the currently existing nodal models and the proposed four-nodal model. The proposed model gives good prediction for all types and combinations of heat loads.

KEYWORDS
displacement ventilation, thermal plume, nodal model, energy efficiency, temperature gradient

1 INTRODUCTION
In displacement ventilation (DV) systems cool air is supplied into the occupied zone of the room near the floor at low velocity and then entrained by buoyant plumes over any warm objects in the room, such as people, computers or heated floor. It results to a two layer room air temperature profile (stratified and mixed). Ideally, the air movements induced by thermal plumes, from low to high level, are capable of transporting heat and pollutants to the layer above the occupied zone, promoting a vertical temperature and contaminants stratification. The transition level between mixed upper layer and stratified layer is called neutral height, which is related to the height where the inflow rate matches the airflow induced by the thermal plumes in the occupied zone. Controlling the neutral height position is one of the most challenging tasks in DV system design.

Increasing the airflow rate raises the neutral height by raising the point where the total thermal plume flow matches the inflow. As a result, two different approaches can be used to control the supply airflow rate: a temperature based design where the design criteria is the room air temperature in the occupied zone; an air quality based design, where the design criteria is air quality and contaminant is let to stratify over the occupied zone. Air quality based design is
typically used in industrial applications where the contaminant stratification plays an important role. In commercial buildings, where cooling is the main issue, temperature based design is the most common method. In this paper the focus is on commercial buildings where thermal comfort is the main concern.

Nowadays, it is more common to use simulation software where the contaminant and temperature gradients are modelled. The most common linear temperature modelling with two room air nodes has been proposed by Mundt (1996).

Several nodal models have introduced that allow for variable slopes between the nodes that compose the temperature profile. Models of three nodes have proposed by Nielsen (2003) and Mateus and da Graça (2015). Multi-zonal models where air flow rates between the nodes are predefined by CFD method have been proposed by Rees and Haves (2001).

Some nodal models are currently available in thermal energy simulation tools. The Rees and Haves model can be used with ESP-r, Mundt and Mateus and da Graça models are implemented in EnergyPlus and Mundt model is also available in IDA ICE.

Several previous DV studies performed physical measurements of selected buoyant flow elements and combinations of heat gains (Kosonen et al. 2016). In this study the novel four-nodal model was proposed to calculate the temperature gradient in a room with various single flow elements and combinations of them. The measurement data was compared with the currently existing nodal models and the proposed four-nodal model.

2 METHODS

In this paper, four design methods are used in validation: linearized temperature gradient (Mundt model), three node model where the height of mixing layer is calculated with plume theory (Nielsen model) and multi-nodal model that allows different slopes for the temperature profile between the nodes and also takes into account radiation between the surfaces (Rees and the proposed novel four-nodal model).

Mundt proposed the linear two-nodal model for the temperature gradient prediction in design. In this model, the radiative energy flux from the floor is balanced by convective heat transfer from the floor surface to the air.

In Nielsen’s model (Nielsen 2003), a linear vertical temperature gradient between floor and the height of mixing layer is predicted with Archimedes number and the type of heat gain. Over the mixing layer, the room air temperature is assumed to be constant. The mixing layer temperature is calculated with energy balance. The floor temperature is determined with specific Archimedes number of the supply air.

The Rees model (Rees and Haves 2001) includes 11 interrelated nodes: 4 room air nodes out of the thermal plume, 4 nodes of the air flowing in the plume and 3 surface nodes representing floor, ceiling and wall temperatures. In addition, Rees model uses 14 flow paths between the nodes with flow rate parameters that have to be pre-determined by a set of rules.

The new four-nodal model predicts room air temperature at four heights: at the height of 0.1 m, at the height of the occupied zone (h_{oc} = 1.2 m), at the height of the mixed layer (h_{mx}). In the cases, when the convective source is located over the occupied zone and there are no other heat gains in the occupied zone, it could be assumed linear temperature distribution between the node at the height of 0.1 m and the mixed layer. In this case the height of the occupied zone can be obtained using the proportion:

\[ h_{oc} = h_{0.1} + (H - 0.1) \cdot (\theta_{oc} - \theta_{0.1}) / (\theta_{mx} - \theta_{0.1}) \]  

where \( \theta_{0.1} \) is the air temperature at the height 0.1 m [°C], \( \theta_{oc} \) is the air temperature at the occupied zone at the height 1.2 m [°C], \( \theta_{mx} \) is the air temperature at the height of mixing layer h_{mx} [°C], H in the height of extract point [m].
The split between convective and radiative heat loads could be assumed to be 50%. The convective heat balance equation for the three nodes could be set by using the following equation:

\[ \rho \cdot c_p \cdot q_v \cdot (\theta_{0.1} - \theta_s) - \rho \cdot c_p \cdot q_v \cdot (\theta_{0.1} - \theta_0) = \alpha_{cf} \cdot A_f (\theta_f - \theta_{0.1}) \]  

(2)

\[ \rho \cdot c_p \cdot q_v \cdot (\theta_{0.1} - \theta_s) + \rho \cdot c_p \cdot q_v \cdot (\theta_{0.1} - \theta_0) - \Phi_{oc} = \alpha_{wl} \cdot A_{wl} \cdot (\theta_{wl} - \theta_{oc}) \]  

(3)

\[ \rho \cdot c_p \cdot q_v \cdot (\theta_{mx} - \theta_{oc}) - \Phi_{mx} = \alpha_{wl} \cdot A_{wl} \cdot (\theta_{wl} - \theta_{mx}) \]  

(4)

\[ \rho \cdot c_p \cdot q_v \cdot (\theta_e - \theta_{mx}) - \Phi_{high} = \alpha_{c} \cdot A_{c} \cdot (\theta_c - \theta_{mx}) + \alpha_{wu} \cdot A_{wu} \cdot (\theta_{wu} - \theta_{mx}) \]  

(5)

where \( q_v \) is the air flow rate [m\(^3\)/s], \( \theta_e \) is the exhaust air temperature [°C], \( \theta_f \) is the average temperature of the floor [°C], \( \theta_{wl} \) is the average temperature of the lateral surface below the mixed layer [°C], \( \theta_{wu} \) is the average temperature of the lateral surface above the mixed layer [°C], \( \theta_c \) is the average temperature of the ceiling [°C], \( \Phi_{oc} \) is the convective heat load in the occupied zone [W], \( \Phi_{mx} \) is the convective heat load between the occupied zone (1.2 m) and the mixed layer [W], \( \Phi_{high} \) is the convective heat load over the mixed layer [W], \( \alpha_{c,c}, \alpha_{c,f}, \alpha_{c, wl} \) and \( \alpha_{c, wu} \) [W/(m\(^2\)·°C)] are convective heat transfer coefficients of the room surfaces: ceiling, floor, wall surfaces below and above the mixed layer \( h_{mx} \). The occupied zone air is assumed to be fully-mixed. The temperature gradient between mixed air node and over the floor node (0.1 m height) is assumed to be linear. Therefore, is the convective heat loads in occupant zone \( \Phi_{oc} \) can be obtained through the proportional relation between the heights of occupant zone and mixing level:

\[ \Phi_{oc} = h_{oc}/h_{mx} \cdot \Phi_{mx} \]  

(6)

Fig.1. Displacement ventilation gradient four-model scheme.

In the four-nodal model of radiation between the surfaces, the wall is divided into the lower (\( A_{wl} \)) and upper (\( A_{wu} \)) surface areas. The radiant heat exchange equations for the room surfaces are introduced in the room surface energy conservation equations, considering the equal impact of the radiative heat transfer to all the surfaces:
$$\alpha_c \cdot (\theta_c - \theta_{\text{mx}}) + \alpha_{w} \cdot (\theta_c - (\theta_f A_f + \theta_{w1} A_{w1} + \theta_{w2} A_{w2})/(A_t - A_c)) = \Phi_t / A_t$$  \hspace{1cm} (7)

$$\alpha_f \cdot (\theta_f - \theta_{0.1}) + \alpha_{r_f} \cdot (\theta_f - (\theta_c A_c + \theta_{f1} A_f + \theta_{f2} A_{f2} + \theta_{w1} A_{w1} + \theta_{w2} A_{w2})/(A_t - A_f)) = \Phi_t / A_t$$ \hspace{1cm} (8)

$$\alpha_{w1} \cdot (\theta_{w1} - \theta_{oc}) + \alpha_{r_{w1}} \cdot (\theta_{w1} - (\theta_c A_c + \theta_{f1} A_f + \theta_{f2} A_{f2} + \theta_{w1} A_{w1} + \theta_{w2} A_{w2})/(A_t - A_{w1})) = \Phi_t / A_t$$ \hspace{1cm} (9)

$$\alpha_{w2} \cdot (\theta_{w2} - \theta_{mx}) + \alpha_{r_{w2}} \cdot (\theta_{w2} - (\theta_c A_c + \theta_{f1} A_f + \theta_{f2} A_{f2} + \theta_{w1} A_{w1} + \theta_{w2} A_{w2})/(A_t - A_{w2})) = \Phi_t / A_t$$ \hspace{1cm} (10)

where $\Phi_t$ is the radiative heat flow [W], $A_t$ is the total area [m$^2$], $A_c$ is the ceiling surface area [m$^2$], $A_f$ is the floor surface area [m$^2$], $A_{w1}$ is the lateral surface area that is below the mixed layer [m$^2$], $A_{w2}$ is the lateral surface area that is above the mixed layer [m$^2$], $\alpha_{r,c}$, $\alpha_{r,f}$, $\alpha_{r,\text{wl}}$ and $\alpha_{r,\text{wu}}$ [W/(m$^2 \cdot ^\circ C$)] are the radiative heat transfer coefficients of the room surfaces: ceiling, floor, wall surface below and above the mixed layer $h_{\text{mx}}$.

When there are $n$ equal strength non-closed positioned plumes, the height of the mixed layer ($h_{\text{mx},p}$) is calculated by:

$$h_{\text{mx},p} = 23.95 \cdot ((q_v/n)^3/\Phi_{cf,p})^{1/5} + h_0$$ \hspace{1cm} (11)

where $\Phi_{cf,p}$ is the convective heat flow from the point source [W], $n$ is the number of heat gains, $h_0$ is the height of virtual origin of a thermal plume. The virtual origin could be calculated using the following equation:

$$h_0 = H_{\text{pl}} + D/3 - 0.8D/(2tg12.5^\circ)$$ \hspace{1cm} (12)

where $H_{\text{pl}}$ is the height of a vertical cylindrical plume source [m], $D$ is the diameter of a vertical cylindrical plume source [m].

The mixing height when the heat loads are on ceiling and floor can be calculated by:

$$h_{\text{mx},h} = q_v/(0.5\cdot 10^{-3}\cdot L^{2/3}\cdot \Phi_{cf,h}^{1/3})^{3/5} - W$$ \hspace{1cm} (13)

where $L$ is the length of horizontal surface [m], $W$ is the width of horizontal surface [m], $\Phi_{cf,h}$ is the convective heat flow from horizontal heat source (ceiling or floor) [W].

The mixing height $h_{\text{mx},w}$ of window heat load can be calculated from the bottom of the window to the level where the flow structure changes from laminar to turbulent $Ra= 7\cdot 10^8$:

$$h_{\text{mx},w} = (Ra \cdot \nu \cdot k/(g \cdot \beta \cdot (\theta_w - \theta_{\text{mx}}) \cdot c_p \cdot \rho \cdot h_{\text{mx},w}^3/(\nu \cdot k)))^{1/3} + H_w$$ \hspace{1cm} (14)

where $Ra$ is Rayleigh number ($Ra = g \cdot \beta \cdot (\theta_w - \theta_{\text{mx}}) \cdot c_p \cdot \rho \cdot h_{\text{mx},w}^3/(\nu \cdot k)$), $g$ is acceleration due to gravity, $\beta$ is the thermal expansion coefficient, $\nu$ is the kinematic viscosity, $\theta_w$ is the window surface temperature, $k$ is thermal conductivity, $\rho$ is air density, $c_p$ is specific heat and $H_w$ is the height of the window.

For the cases with combination of flow elements, the mixing height is adjusted using the weight factors for all the heat loads:

$$\Phi_{cf} = \Phi_{cf,p} + \Phi_{cf,h} + \Phi_{cf,w}$$ \hspace{1cm} (15)

$$h_{\text{mx}} = h_{\text{mx},p} \cdot \Phi_{cf,p}/(h_{\text{mx},h} \cdot \Phi_{cf,h}/\Phi_{cf} + h_{\text{mx},w} \cdot \Phi_{cf,w}/\Phi_{cf})$$ \hspace{1cm} (16)
3 RESULTS

3.1 Single buoyant flow element cases.
The results of the measurements and the calculations with the selected models are shown for the single buoyant flow element in Fig 2.

![Fig 2](image)

Fig 2. Measured and calculated room air temperature profiles of single buoyant flow elements: a) window heat load (520W) and b) 6 occupants (450 W).

The room air temperature profiles of window flow elements are quite linear whereas temperature profiles of close to floor level centralized heat gains are far from linear. With those heat gains, the major part of the room air temperature gradient happens already in the occupied zone. Therefore, unlike linear two nodal models the multi-nodal models give an accurate temperature gradient prediction for the cases with occupant flow elements. At the same time, in a case with window heat loads the predictions of linear Mundt model and four-nodal models are quite accurate.

3.2. Combinations of the different heat loads.
The results for the combination cases are presented in Fig 3.

![Fig 3](image)

Fig 3. Measured and calculated room air temperature profiles of combination cases: a) 10 occupants (750W), window heat loads (520W), lighting (232W) b) 10 occupants (750 W), ceiling heat loads (466W).
With combinations cases when window load is combined with lights and person loads, the multi-nodal models gives good estimation of room air temperature profile. However in the case with high level loads in ceiling, only Rees and the proposed four-nodal models can give the closest temperature gradient prediction. However, Rees and Haves model doesn’t predict accurately the temperature gradient below 1.2 m. Two-nodes linear models are not able to estimate room air temperature profiles. Those models predict too low room air temperatures in the occupied zone.

4 DISCUSSION
Two-nodal models are not able to predict the vertical temperature gradient with typical heat loads. The accuracy of the earlier proposed multi-nodal models is acceptable with the heat gains that are located at the low level of the occupied zone. When the heat gains are located at high level (warm ceiling) or linear type (warm window), the accuracy of vertical air temperature estimation differ significantly from the measurements. As a result, thermal comfort in the occupied zone is lower than predicted. It means that the set thermal conditions have not really fulfilled when the previously presented simplified calculation methods have been used. With the novel proposed four-node model, it is possible to improve the temperature gradient prediction especially with window and high-level located heat loads.

5 CONCLUSIONS
In this study, the performance of a displacement ventilation system is studied under a variety of load conditions. The single buoyant flow elements from occupants and heated window as well as heat load combinations were measured in a simulated office room. The measurement data were used to validate a selected set of models of DV systems and the proposed novel four-node model. Linear models are not able to predict the temperature gradient with displacement ventilation. The Rees and Nielsen multi-nodal models consider stratification in the occupied zone and give an accurate gradient estimation for the cases with low-level flow elements. However, these models could not predict the temperature gradient in the cases with window heat loads. The proposed four-nodal models can give better prediction than other models for all the presented cases.

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7 REFERENCES