NUMERICAL STUDY OF PARTICLE DISTRIBUTION IN THE WAKE OF PARTICLE-LADEN AIR FLOWS PAST A RECTANGULAR BUILDING USING DISCRETE VORTEX METHOD

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ABSTRACT
A numerical investigation on particle distribution in the wake of particle-laden airflows past a rectangular building at Reynolds number of $10^7$ is presented. The Discrete Vortex Method is employed to evaluate the unsteady airflow fields and a Lagrangian approach is applied for tracking individual solid particles. A dispersion function is defined to represent the dispersion scale of the particle. The wake vortex patterns and the distributions of particles with Stokes numbers, $St$, of 0.01, 0.1, 1.0 and 10 are obtained. The results show that the particle distribution in the wake of the rectangular building is closely related to the particle’s Stokes number and the structure of wake vortices: (1) very small sized particles with $St=0.01$ behave like gas elements and can distribute both in the vortex core and around the vortex periphery, whereas the intermediate sized particles with $St=1.0, 10$ can not enter the vortex cores and concentrate near the peripheries of the vortex structures due to the centrifugal forces acting on the particles by the wake vortices; (2) the dispersion intensity of particle decreases as $St$ increases from 0.01 to 10. This work will be helpful in predicting and analyzing particle concentrations around rectangular buildings.

INDEX TERMS
Particle-laden airflows, Rectangular building, Discrete Vortex Method, Particle distribution

INTRODUCTION
Unsteady particle-laden airflows (such as dust-polluted airflows from process operations in industries, vehicle-induced dust-laden airflows, and dust storms etc.) past rectangular buildings (rectangular cylinders) at high Reynolds numbers are a common occurrence. Because gas flows past a rectangular cylinder are characterized by the development of wake vortices, understanding the mechanism of particle distribution in the wake of a rectangular cylinder is crucial for predicting particle pollution around rectangular buildings.

Up to now, no numerical or experimental studies on particle dispersion in gas-particle flows past a rectangular cylinder at high Reynolds number have been reported. D.J. Brandon and S.K. Aggarwal (2001) conducted a numerical investigation on particle dispersion in the wake of a rectangular cylinder and found that the particle dispersion pattern was determined by the particle’s stokes number. However, D.J. Brandon and S.K. Aggarwal in their work only investigated the laminar and transitional regimes (for the range of Reynolds numbers) and did not quantitatively study the dispersion intensity of particles.

The present work aims to numerically investigate the distributions of different sized particles in the wake of a rectangular building at high Reynolds number. Here, we study the two-dimensional, dilute gas-particle flows past a rectangular cylinder at Reynolds number of $10^7$. In the numerical procedure, the Discrete Vortex Method is used to compute the unsteady airflow fields and a Lagrangian approach is applied for tracking individual particles; In the simulation, the change in the particle distribution pattern and in the intensity of particle dispersion with the Stokes number are explored.

BASIC EQUATIONS AND NUMERICAL METHOD
Unsteady gas flow field
We use the Discrete Vortex Method (Wu wenquan and F sisto 1987, A.J.Chorin 1973) to evaluate the unsteady airflow fields numerically.

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For two-dimensional, viscous, homogeneous incompressible gas flows past a rectangular cylinder at high Reynolds number, the vorticity transport equation is given as

$$\frac{D \omega}{Dt} = \nu \nabla^2 \omega$$  \hspace{1cm} (1)

where $D / Dt$ is the constitutive derivative, $t$ is the time, $\nu$ is the kinematic viscosity of the gas, the vorticity $\omega$ is expressed by

$$\omega(x, y, t) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$  \hspace{1cm} (2)

where $u$ and $v$ are the gas velocity components in $x$- and $y$- directions, respectively.

The stream function $\psi$ is related to the velocity components through

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$  \hspace{1cm} (3)

which satisfies the continuity equation. Combining Eqs. (2) and (3) yields

$$\nabla^2 \psi = -\omega$$  \hspace{1cm} (4)

The Poisson’s equation (4) enables one to determine $\psi$ from $\omega$.

In the Discrete Vortex Method, the continuous vorticity field is represented by the sum of a large number of discrete vortex blobs. During the simulation, the discrete vortex blobs are tracked using a grid-free vortex-tracing algorithm, once the position and strength of the vortex blobs are determined, the whole velocity field can be obtained by using the Biot-Savart Law.

**The Lagrangian equation of motion for a particle**

In the present simulation, the following assumptions are made:

1. The gas-particle two-phase flow is dilute, i.e., the particle-to-particle collisions can be neglected (R. Ishill et al. 1989), and the gas flow is not affected by the particles;
2. The particles are spherical and not rotating;
3. The flow is two dimensional, and the flow plane is perpendicular to the gravity.

Then, the Lagrangian equation of motion for a particle we used is

$$\pi d_p^3 \rho_p \frac{dV_p}{dt} = 3 \pi d_p \rho \left( V - V_p \right) \frac{dV_p}{dt} \left( V - V_p \right) + \pi d_p^3 \rho \frac{dV_p}{dt} \left( V - V_p \right) + \frac{d_p^3}{2} \rho \sqrt{\pi \nu} \int_0^\tau \left( \frac{dV_p}{dt} - \frac{dV_p}{d\tau} \right) d\tau$$

$$+ 1.615 d_p^2 \mu \rho \left[ (V - V_p) \frac{dV_p}{dt} \right] + \frac{d_p^3}{2} \rho \left( \frac{DV_p^V}{dt} \right)$$  \hspace{1cm} (5)

where $\rho_p$ and $\rho$ are the particle and gas density, respectively, $d_p$ is the particle diameter, $\mu$ is the dynamic viscosity of the gas, $V$ and $V_p$ are the instantaneous velocity of the gas and the particle, respectively, $f$ is the modification factor for the Stokes drag coefficient, $d / dt$ is the temporal derivative along the discrete particle trajectory and $D / Dt$ is the temporal derivative along the gas motion.

The terms in the right-hand side of Equation (5) are referred to as drag, added-mass, Basset, saffman, and fluid (due to the fluid pressure gradient and viscous stresses) forces.

The particle Reynolds number is defined by
\[ \text{Re}_p = \frac{|V - V_p|d_p}{V} \]  

(6)

\( f \) related to the particle Reynolds number is given below

\[ f = 1 \quad (\text{Re}_p < 1), \quad f = \left(1 + \text{Re}_p \right)^2 / 6 \quad (1 \leq \text{Re}_p \leq 1000), \quad f = 0.44 \frac{\text{Re}_p}{24} \quad (\text{Re}_p > 1000) \]

Let \( L \) be a characteristic length, \( U \) be a characteristic velocity, and \( T = L/U \) be the characteristic time, we set \( x' = x' L, \quad y' = y' L, \quad \nabla = \nabla / L, \quad \dot{V} = \dot{V} U, \quad \dot{V}_p = \dot{V}_p U, \quad t' = t' L/U \).

After substituting and dropping primes, Equation (5) takes the following dimensionless form

\[ \left(1 + \frac{1}{2\alpha}\right)\frac{dV}{dt} = \frac{f}{St} \left(\frac{V - V_p}{St}\right) + \frac{\alpha}{St} \left(\frac{dV}{dt} + \nabla \cdot V + \frac{1}{2\alpha} \frac{dV}{dt}\right) + \frac{3}{\sqrt{2\pi}} \sqrt{\frac{\alpha}{St}} \left(\frac{d\dot{V}}{dt} - \frac{dV}{dt}\right) \]

\[ + \frac{1.615 \sqrt{2}}{\pi} \sqrt{\frac{\alpha}{St}} \left[\frac{\dot{V}}{2} \left(\frac{\partial V}{\partial x} + \frac{\partial u}{\partial y}\right) - \frac{\partial u}{\partial y} \right] \tag{7} \]

where \( \alpha = \rho / \rho_p \), \( St \) is the Stokes number for particles, which is defined as

\[ St = \frac{\rho_p d_p^2}{18 \mu} \frac{1}{L/U} \]  

(8)

Numerical procedure
The unsteady gas flow field is first evaluated by use of the Discrete Vortex Method; based on the obtained unsteady gas flow field, the Lagrangian equations of motion for particles are then solved. These two solution phases are coupled at each time step, such that the effect of unsteady gas flow on particles can be effectively accounted for in the simulation.

NUMERICAL RESULTS AND ITS ANALYSIS
Calculating conditions
We use the above numerical method to simulate the two-dimensional, dilute gas-particle flows past a rectangular building. Fig.1 shows the calculating region. The \( x \)-axis is parallel to the undisturbed upstream and the flow plane is perpendicular to gravity. The flow is from left to right; at time \( t = 0 \) the flow is started with constant nondimensional velocity of magnitude 1.0 in the \( x \)-direction. The kinematic viscosity of the gas is \( 1.5 \times 10^{-6} m^2/s \). The Reynolds number defined by the gas velocity upstream of the rectangular building \( \infty \) and the width of the rectangular cylinder is \( 10^7 \). \( \alpha = \rho / \rho_p = 0.00061 \). Four different particles with Stokes numbers, \( St \), of 0.01, 0.1, 1.0 and 10.0 are calculated. The time step \( \Delta t \) of the simulation of gas flow is set to be 0.018, which is small enough so that the Lagrangian equations of motion for particles can be integrated over this time step.

![Figure 1. Calculating region](image)

At each time step, 30 spherical particles of monosize distribution are released into the gas flow field from the
upstream boundary $x = -2$ where the particle velocity is set to be $(V_w, 0)$, and the initial position of the particle is determined by using random numbers.

In order to study the dispersion scales of particles with different Stokes numbers, the dispersion function in the $y$ direction is used here, which is defined as (Ling W et al. 1998)

$$D_y(t) = \left( \frac{1}{N_p} \sum_{i=1}^{N_p} [Y_y(t) - Y_{m}(t)]^2 \right)^{1/2} \tag{9}$$

where $N_p$ is the total number of particles in the calculating region at time $t$, $Y_y(t)$ is the displacement of the $i$th particle in the $y$ direction from time $t - \Delta t$ to $t$, $Y_{m}(t)$ is the mean value of particle displacement in the $y$ direction from time $t - \Delta t$ to $t$.

**Numerical results**

At time $t=16.2$, the vortex patterns and the distributions of particles with various Stokes numbers are shown in Figs. 2(a) through 2(d). From these figures it is clear that the particle distribution pattern in the wake of the rectangular building is characterized by the particle’s Stokes number and by the wake vortex structures.

Because the particles with very small Stokes numbers have very small inertia effects and follow closely the gas elements, the particles with $St=0.01$ can distribute both in the vortex core and around the vortex periphery as shown in Fig. 2(a).

When the Stokes number of particles is increased from 0.01 to 0.1, the particles distribute as shown in Fig. 2(b). Comparing with the distribution pattern for particles with $St=0.01$ (Fig. 2(a)), the region around the vortex cores, where few particles exist, becomes wider. This is due to the increment in the centrifugal force acting on the particle by the wake vortices.

For intermediate-$St$ particles, the particle response time is of the same order of magnitude as the time scale of the gas flow. Due to the strong centrifugal force acting on the particle by the vortices, the intermediate sized particles with $St=1.0$ and 10.0 cannot enter the vortex core and distribute around the vortex periphery as depicted in Figs 2(c) to 2(d).
Figure 3 shows the time series of dispersion function of particles with Stokes numbers ranging from 0.01 to 10.0. It is evident from this figure that the dispersion scale of particle decreases as St is increased from to 10.0.

CONCLUSIONS

The two-dimensional, dilute gas-particle two-phase flows past a rectangular building at Reynolds number of $10^7$ is numerically simulated using the Discrete Vortex Method. The wake vortex patterns, the distributions and the time series of dispersion functions of particles with Stokes numbers, $St$, of 0.01, 0.1, 1.0 and 10 are obtained. The numerical results show that the particle distribution in the wake of a rectangular building is closely related to the particle’s Stokes number and the structure of the wake vortices: (1) very small sized particles with $St=0.01$ behave like gas elements and can distribute both in the vortex core and around the vortex periphery; (2) the regions around the vortex cores, where few particles exist, become wider as the stokes number for particles is increased from 0.01 to 1.0; (3) due to the increment in the centrifugal force acting on the particle by the wake vortices and the particle’s inertia, the particles with $St=1.0$ and 10.0 can not enter the vortex cores and concentrate near the peripheries of the vortex structures; (4) in the wake of the rectangular building, the dispersion intensity of particle decreases precipitously as $St$ is increased from 0.01 to 10.0.

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